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硕士学位论文

基于收益率与极值的波动率拟最大似然估计

Simulated Maximum Likelihood Estimation of Volatility Based on Return and Extreme values

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摘　要

在金融市场中，波动率是金融风险和资产定价研究领域的核心。真实的波动率参数通常不能被直接观测，需要通过代理变量进行估计。其中，较常用代理变量是基于高频数据的已实现波动率。然而在某些情况下，高频数据的获得存在一定的困难。相比之下，日度的开盘\收盘\最高\最低的数据结构更为常见。本文研究了利用开盘\收盘\最高\最低数据信息的波动率估计方法。

由于包含开盘\收盘\最高\最低价的似然函数无解析解，本文对包含四个价格的似然函数进行近似；并采用序贯蒙特卡洛抽样方法来估计近似似然函数，进一步可以得到波动率的拟最大似然估计。在使用序贯蒙特卡洛方法估计近似似然函数的过程中，本文选择截断分布作为抽样分布，提高了抽样效率。通常序贯蒙特卡洛抽样法的随机性，会导致估计的近似似然函数不平滑。针对该问题，本文保留了随机数种子，并利用了分布函数的反函数法和相同的随机数种子来抽样，从而保证估计的似然函数平滑性。模拟的结果显示：在抽样间隔足够小时，本文的估计量效果好于基于收益率和基于极差的估计方法。证明更多的价格信息可以有效改善估计效果。

关键词：扩散过程；序贯蒙特卡洛抽样；波动率

Abstract

In financial market, the volatility of the price is an important and practical issue in pricing assets and managing risk. However, the latent volatility is unobservable, and it is necessary to use a volatility proxy. Based on the high-frequency data,the realized volatility could be an accurate estimator. However in some markets high-frequency data is not always available. The daily open/close/ high/low prices are more common than high-frequency data .This paper focus on the estimation of volatility only taking the advantage of daily open/close/high/low prices.

Because there is no analytic solution to the likelihood function containing open/close/high/low information, this paper try to get the approximation of the exact likelihood function, and use the Sequential Monte Carlo sampling method to estimate the approximation, further can get simulated maximum likelihood estimation of volatility. In the process of estimating the approximation of likelihood function, the truncated distribution is selected as the sampling distribution in Sequential Monte Carlo method to improve sampling efficiency. In general ,the random nature of the sampling process can result in the unsmooth of estimated likelihood function . In order to solve this problem, this paper retains the random seeds,and use the inverse distribution function method and the same random seeds for sampling. The simulation results show that the estimation efficiency of this paper is better than the method based on the range or return. It proves that more price information can effectively improve the estimation.

Keywords: Stochastic Diffusion; Sequential Monte Carlo; Volatility

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